AP Rev. WS #8

1a) f' is decreasing on $1.7 < x < 1.9$ so f'' is negative. f will be concave down.

1b) Possible extrema of f where $f' = 0$ or @ endpts $x = 0$ or $x = 3$
Absolute max on a <u>closed</u> interval
Check endpts and where f' goes from $+$ (f increasing) to $-$ (f decreasing)

$0 = e^{-x/4} \sin x^2$

$e^{-x/4} \neq 0 \quad \sin x^2 = 0$

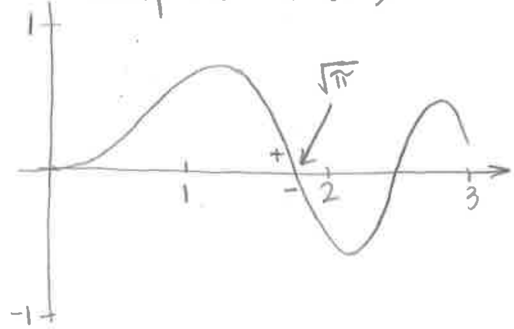
$x^2 = \pi n$

$x = \pm\sqrt{\pi n}$

on $[0, 3] \rightarrow x = \sqrt{\pi}$

$x \approx 1.77$

Graph of $f'(x)$



Absolute Max @ $x = \sqrt{\pi}$

Endpts?
$x = 0 \rightarrow f'+$, so f increasing on $(0, \sqrt{\pi})$; so $f(0) < f(\sqrt{\pi})$

To confirm that $f(3) < f(\sqrt{\pi})$ it would be great to know the equation for $f(x)$! Next term...

2a) $f'(22) = -3$ Cal/min^2 from $m = \frac{15-3}{20-24} = \frac{12}{-4}$

b) f increasing at its greatest rate: find the steepest slope of f (highest value)

so looking for max of f'
find possible extrema of f''

$f'(t) = -\frac{3}{4}t^2 + 3t$ for $0 \leq t \leq 4$ Max of $f'(t)$ where f'' goes from $+$ to $-$

$f''(t) = -\frac{3}{2}t + 3$

$0 = -\frac{3}{2}t + 3$

$t = 2$ critical #

f'': 0 ⟵ + ⟶ 2 ⟵ − ⟶ 4, t

confirms that f is increasing at its greatest rate at $t = 2$

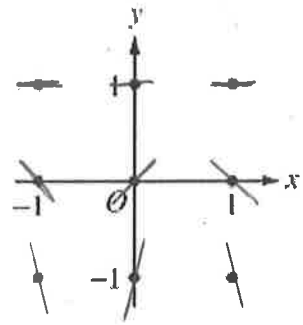
3.

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$. $\frac{dy}{dx} = 0$ @ $y = 1$

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

x	y	$\frac{dy}{dx}$
1	0	$\cos \pi = -1$
1	−1	$4 \cos \pi = -4$
0	0	$\cos 0 = 1$
0	−1	$4 \cos 0 = 4$
−1	0	$\cos(-\pi) = -1$
−1	−1	$4 \cos(-\pi) = -4$



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

$c = 1$

4.

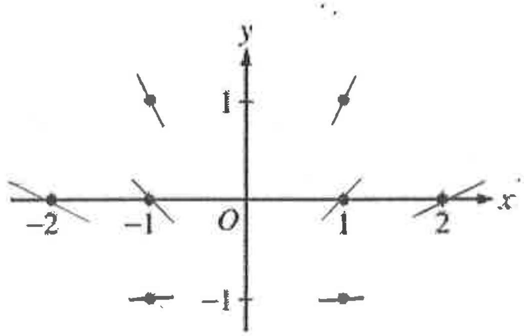
so leave the y-axis blank.

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$. $\frac{dy}{dx} = 0$ @ $y = -1$

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)

x	y	$\frac{dy}{dx}$
−2	0	$-\frac{1}{2}$
−1	0	−1
−1	1	−2
1	0	1
1	1	2
2	0	$\frac{1}{2}$



5. a) $g'(x) = ae^{ax} + f'(x) \longrightarrow g'(0) = ae^{a(0)} + f'(0) = a - 4$

$g''(x) = a^2 e^{ax} + f''(x) \rightarrow g''(0) = a^2 e^{a(0)} + f''(0) = a^2 + 3$

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2,\ f'(0) = -4,\ \text{and}\ f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$. $\rightarrow h(0) = (\cos 0) f(0) = (1)(2)$

point of tangency $(0, 2)$

$h'(x) = \cos(kx) f'(x) + (-\sin(kx))(k)$

$h'(x) = f'(x)\cos(kx) - k\sin(kx)$

$\boxed{y = -4x + 2}$

slope $= h'(0) = f'(0)\cos(0) - k\sin(0) \rightarrow h'(0) = -4(1) - 0$